Proposal of an experimentally accessible measure of many-fermion entanglement

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We propose an experimentally accessible measure of entanglement in many-fermion systems that characterizes interaction-induced ground state correlations. It is formulated in terms of cross-correlations of currents through resonant fermion levels weakly coupled to the probed system. The proposed entanglement measure vanishes in the absence of many-body interactions at zero temperature and it is related to measures of occupation number entanglement. We evaluate it for two examples of interacting electronic nanostructures.

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Entanglement is a distinguishing feature of quantum mechanics [1] and it is regarded as the defining resource for many of its modern applications such as quantum communication and quantum computation. Despite intensive efforts, however, a thorough understanding of entanglement beyond the bipartite setting has not yet been reached. Its characterization in systems of indistinguishable particles has proven particularly challenging. A key insight into the problem has been that entanglement is an observer dependent concept [2, 3, 4]. Different characterizations of it are therefore possible and they may prove useful in different situations. A number of entanglement measures for systems of indistinguishable fermions have thus been proposed [3, 4, 5, 6, 7, 8] with focus on different aspects of the phenomenon. Studies of the entanglement of many-particle systems have already resulted in several valuable new insights [9, 10, 11] and they promise to continue to contribute to our understanding of complex phenomena such as quantum phase transitions [9, 10].

Both, the challenge as well as the fascination of many-particle systems originate from many-body interactions. Measures of the entanglement that is induced by these interactions [8] should therefore characterize such systems in a particularly instructive way. Their theoretical evaluation for systems of interacting fermions faces, however, severe limitations. In space dimensions larger than one many of these systems are neither accessible by any analytical nor by any numerical [12] tools available to date. Experiments, either directly on the systems of interest or on quantum simulators, that is artificial systems with equivalent Hamiltonians, can help to remedy the situation. In this Letter we therefore propose an entanglement measure for many-fermion systems that can be evaluated with existing experimental techniques and that quantifies interaction-induced ground state correlations. It is based on a generalization of the notion of entanglement put forward in Refs. [3, 4]. We show that it is closely related to measures of occupation number entanglement [6, 7] in case one can define a noninteracting counterpart of an interacting system, but that it generalizes the concept. Being experimentally accessible the proposed entanglement measure is a novel probe of interaction-induced ground state correlations in fermionic systems.

To exemplify the proposed notion of entanglement we apply it to electronic nanostructures. Due to their small dimensions nanoscale conductors are typically strongly affected by many-body interactions and they are thus natural candidates for such studies. In fact, the characterization of interaction-induced correlations is one of the main challenges that these systems pose. Suitable experimental probes have been identified for some of these correlations, for instance shot noise to detect fractional effective charges in quantum Hall systems [13]. For many nanostructures of interest, however, standard experimental techniques such as current correlation measurements are insensitive to interaction-induced correlations [14]. The proposed many-fermion entanglement, in contrast, promises to be a systematic probe of such correlations in these systems. It involves the measurement of cross-correlations of electrical currents through resonant levels that are weakly coupled to the studied system. Both required ingredients, tunable resonant levels in the form of quantum dots as well as the ability to perform cross-correlation measurements of electrical currents [15], have been demonstrated experimentally.

Formalizing the observer dependence of entanglement Barnum, Knill, Ortiz, and Viola have introduced the concept of a "generalized entanglement" [3]. It is defined in terms of a set of experimentally accessible observables $\mathfrak{h} = \{A_1 \dots A_n\}$ rather than a spatial partitioning of a system. It reduces to the conventional measures of bipartite entanglement as one chooses for $\mathfrak{h}$ the set of all operators A_j that act locally in two partitions of a system. The degree of entanglement of a quantum state in this formulation is determined by the expectation values of the observables A_j in that state. A state is defined to be entangled if it does not produce extremal expectation values. This can be motivated by the bipartite case where entanglement induces mixed reduced density matrices for the subsystems and thus renders expectation values of local operators generically non-extremal. More formally, a quantum state with density matrix ρ_α

is represented by a linear functional λ_α on $\mathfrak{h}$ that gives the expectation values $\lambda_\alpha(A_j) = \mathrm{tr}\,\rho_\alpha A_j$. We employ a formulation of generalized entanglement based on convex cones [3] and consider the convex cone C_n of all linear combinations $\lambda = \sum_\alpha p_\alpha \lambda_\alpha,\ p_\alpha \geq 0$ of functionals $\lambda_\alpha \in \{\lambda_g, \lambda_0, \ldots, \lambda_n\}$. The λ_α are defined by density matrices ρ_α of the ground state (ρ_g) and other experimentally accessible states (ρ_j) of a quantum system. Elements of C_n with $\sum_\alpha p_\alpha = 1$ are referred to as states. States that cannot be expressed as linear combinations of other states are extremal in C_n and imply extremal expectation values. They are thus called pure. For a state $\lambda \in C_n$ the degree of generalized entanglement $\mathcal{E}_n$ is defined through a Schur-concave function S as [16]

$$\mathcal{E}_n(\lambda) = \inf\{S(\mathbf{p}) | \lambda = \sum_\alpha p_\alpha \lambda_\alpha \ \text{ with } \lambda_\alpha \text{ pure states}\}. \tag{1}$$

We take $S(\mathbf{p}) = -\sum_\alpha p_\alpha \ln p_\alpha$, the Shannon entropy. $\mathcal{E}_n$ depends on both, the set of accessible observables $\mathfrak{h}$ as well as the set of states λ_α that define C_n.

Being formulated directly in terms of expectation values this generalized concept of entanglement allows us to define the advertised experimentally accessible measure of many-fermion entanglement. We introduce it for electronic systems $\mathcal{S}$, but it has an obvious extension to arbitrary fermionic systems. We choose electrical currents as the observables A_j. Typically electrical currents are correlated even in the absence of electron-electron interactions [17]. To selectively characterize the quantum correlations induced by many-body interactions we therefore consider the setup depicted in Fig. 1: $\mathcal{S}$ is weakly coupled to at least two resonant levels j with resonance energies ϵ_j. For small level broadenings $\Gamma_k, \Gamma_l \ll |\epsilon_k - \epsilon_l|$ the currents through two levels j, k are then uncorrelated in the absence of interactions. Γ_j are due to a coupling of the levels to two macroscopic leads ("reservoirs") each, corresponding to the Hamiltonian

$$H = H_0 + \sum_j \epsilon_j d_j^\dagger d_j + \left[v_j' \psi_j^\dagger d_j + d_j^\dagger (v_j a_j + \tilde{v}_j \tilde{a}_j) + h.c. \right]. \tag{2}$$

Here, H_0 is the Hamiltonian of $\mathcal{S}$ and the reservoirs when decoupled, $\psi_j^\dagger$ creates an electron in $\mathcal{S}$ in the mode that is contacted by level j, $d_j^\dagger$ creates an electron on level j, and $a_j^\dagger$, $\tilde{a}_j^\dagger$ create electrons in the reservoirs attached to it. $\mathcal{S}$ is assumed to be initially uncoupled from the reservoirs and in its ground state. After the coupling has been turned on it is still arbitrarily close to its ground state in the limit $v_j' \to 0$ that we consider. We thus call the state of the coupled setup $|g\rangle$. We refer to the currents into the reservoirs coupled to d_j as I_j and $\tilde{I}_j$ respectively. We assume idealized noninteracting reservoirs with infinite bandwidth that are either completely occupied by electrons or entirely empty, such that $I_j \propto \langle g| \overline{f_j^\dagger f_j} |g\rangle$. We have $f_j = d_j$ if the reservoirs coupled to j are empty

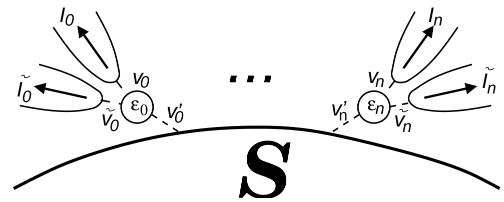

FIG. 1: System $\mathcal{S}$ contacted by resonant levels $j = 0 \ldots n$ that emit electron currents I_j, $\tilde{I}_j$ into two reservoirs each.

and $f_j = d_j^\dagger$ if they are occupied. Due to the Pauli principle I_j and $\tilde{I}_j$ are perfectly anticorrelated at equal times: $\langle \tilde{I}_j(t) I_j(t) \rangle_\mathrm{s} = 0$ while $\langle I_j \rangle, \langle \tilde{I}_j \rangle \neq 0$ (we abbreviate $\langle g| \ldots |g\rangle$ by $\langle \ldots \rangle$; $\langle AB \rangle_\mathrm{s} = \langle AB + BA \rangle / 2$), and

$$\langle \tilde{I}_j(t) I_j(t + \delta t) \rangle_\mathrm{s} = \mathcal{O}(\Gamma_j \delta t). \tag{3}$$

The levels d_j may be implemented by quantum dots. They are assumed to be noninteracting which can be realized by small quantum dots in a large magnetic field that can be occupied by single electrons only. The leads coupled to the resonant levels can be emptied of and filled with electrons by applying bias voltages.

We here introduce $\mathcal{E}_1$, a single observable version of the proposed entanglement measure. For that we choose $\mathfrak{h}_1$ to consist of two operators: $A_0 = 1$ and $A_1 = Q_1 = \int_\tau^\infty dt\, \exp[-\gamma(t - \tau)] I_1(t)$. We consider the regime $\Gamma_0, \Gamma_1 \ll \gamma \ll |\epsilon_0 - \epsilon_1|$ and $\tau^{-1} \approx \gamma$. We define C_1 through the states $\rho_g = |g\rangle\langle g|$ and $\rho_j = \langle g| \overline{f_j^\dagger f_j} |g\rangle^{-1} \, \overline{f_j |g\rangle\langle g| f_j^\dagger}$ ($j = 0, 1$). The overbar denotes a time integral, $\overline{\ \ldots\ } = \int_0^\tau dt\, \exp(-\gamma t)\ \ldots$. The states ρ_α are normalized such that $\lambda_\alpha(A_0) = 1$ for all α. $\mathcal{E}_1$ of $|g\rangle$, a measure of the entanglement of the ground state of $\mathcal{S}$, is now defined through Eq. (1) as the generalized entanglement of λ_g within C_1 in the sense of Refs. [3, 4].

The form of the current expectation values $\langle I_j \rangle \propto \langle g| \overline{f_j^\dagger f_j} |g\rangle = \mathrm{tr}\, \overline{f_j |g\rangle\langle g| f_j^\dagger}$ suggests that one can project onto the states ρ_0 and ρ_1 by current measurements. One can show that accordingly the functionals λ_j can be expressed through current correlators, $\lambda_j(A_1) = \langle \tilde{Q}_j^p Q_1 \rangle_\mathrm{s} / \langle \tilde{Q}_j^p \rangle$ with $\tilde{Q}_j^p = \int_0^\tau dt\, \exp(-\gamma t) \tilde{I}_j(t)$ [18]. The λ_α that define C_1 expressed in terms of observables are summarized in table I. We have $\lambda_\alpha \geq 0$ since all the

state λ	density matrix	$\lambda(A_0)$	$\lambda(A_1)$
λ_g	$\|g\rangle\langle g\|$	1	$\langle Q_1 \rangle$
λ_0	$\langle g\| \overline{f_0^\dagger f_0} \|g\rangle^{-1}\, \overline{f_0 \|g\rangle\langle g\| f_0^\dagger}$	1	$\langle \tilde{Q}_0^p Q_1 \rangle_\mathrm{s} / \langle \tilde{Q}_0^p \rangle$
λ_1	$\langle g\| \overline{f_1^\dagger f_1} \|g\rangle^{-1}\, \overline{f_1 \|g\rangle\langle g\| f_1^\dagger}$	1	$\langle \tilde{Q}_1^p Q_1 \rangle_\mathrm{s} / \langle \tilde{Q}_1^p \rangle = 0$

TABLE I: State functionals λ_j that define C_1 together with the density matrices that induce them for $\Gamma_1 \tau \to 0$.

relevant currents are positive. Also, according to Eq. (3) $\lambda_1(A_1) = 0$ in our limit $\Gamma_1 \tau \to 0$. One can argue [18],

that one generically further has $\lambda_0(A_1) \geq \lambda_g(A_1)$. Hence λ_g is typically not extremal in C_1 and in that case has the unique representation $\lambda_g = (\lambda_0 + \alpha\lambda_1)/(1+\alpha)$ with $\alpha = \lambda_0(A_1)/\lambda_g(A_1) - 1 \geq 0$. With Eq. (1) we find from this the ground state entanglement of $\mathcal{S}$

$$\mathcal{E}_1(\lambda_g) = -\frac{\alpha}{1+\alpha}\ln\frac{\alpha}{1+\alpha} - \frac{1}{1+\alpha}\ln\frac{1}{(1+\alpha)}. \tag{4}$$

α is given by a normalized irreducible current correlator,

$$\alpha = \frac{\langle\langle \tilde{Q}_0^p Q_1\rangle\rangle_{\mathrm{s}}}{\langle\tilde{Q}_0^p\rangle\langle Q_1\rangle}, \tag{5}$$

where $\langle\langle AB\rangle\rangle_{\mathrm{s}} = \langle AB\rangle_{\mathrm{s}} - \langle A\rangle\langle B\rangle$. The correlator α has a well-defined limit for $v_j' \to 0$, when it characterizes the ground state of $\mathcal{S}$ unperturbed by the measuring apparatus [18]. In our limits it vanishes in the ground state of any system of noninteracting electrons. Thus $\mathcal{E}_1$ selectively characterizes ground state entanglement that is induced by many-body interactions. It is determined by cross-correlations of electrical currents,

$$\alpha = \int \frac{d\omega}{2\pi}\frac{e^{-i\omega\tau} - e^{-\gamma\tau}}{1-e^{-\gamma\tau}}\frac{\gamma^2}{\omega^2+\gamma^2}\frac{\langle\langle \tilde{I}_0 I_1\rangle\rangle_\omega}{\langle\tilde{I}_0\rangle\langle I_1\rangle}, \tag{6}$$

where $\langle\langle \tilde{I}_0 I_1\rangle\rangle_\omega = \int dt\, \exp(i\omega t)\langle\langle \tilde{I}_0(0) I_1(t)\rangle\rangle_{\mathrm{s}}$. There are alternative ways to measure α or similar correlators, for instance using low temperature reservoirs with variable chemical potential as energy filters instead of resonant levels. This and a regime of low-frequency detection $\gamma \ll \tau^{-1} \ll \Gamma_j$ will be discussed elsewhere [18].

An intuitive interpretation of $\mathcal{E}_1$ in terms of ground state properties is possible if the electron-electron interactions in $\mathcal{S}$ are absent during the current measurement. This can be achieved by switching off the interactions at a time $\tau_s < 0$ suddenly, such that $\mathcal{S}$ remains in its interacting ground state. At small γ, such that on the scale γ all ground state amplitudes depend only weakly on the single-particle energies involved, the normalized correlator α as defined in Eq. (5) then takes the form

$$\alpha = \frac{\langle\langle n_0 n_1\rangle\rangle}{\langle n_0\rangle\langle n_1\rangle} \quad \text{for } f_j = d_j. \tag{7}$$

For a finite-sized system in the limit $\tau_s\Gamma_j \to -\infty$ $n_j = \psi^\dagger_{k_j}\psi_{k_j}$ is the occupation number of noninteracting eigenmodes ψ_{k_j} of $\mathcal{S}$ with quantum numbers k_j and noninteracting energies $\epsilon_{k_j} = \epsilon_j$ [19]. $\mathcal{E}_1$ in that case thus quantifies correlations between fermion occupation numbers in the ground state of $\mathcal{S}$, similarly to the entanglement measures proposed in Refs. [6, 7, 8]. Such correlations are induced by interactions typically through the creation of particle-hole pairs. Extensions of $\mathcal{E}_1$ that capture ground state entanglement due to multiple particle-hole pairs will be discussed in [18]. A sudden switching off of interactions is possible in cold gases of fermionic atoms

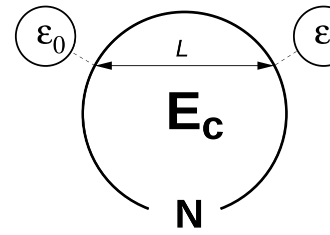

FIG. 2: Large quantum dot opened up by N conduction channels coupled to two resonant levels implemented by small dots.

[20]. Electronic systems, however, may be regarded as noninteracting during the current measurement only in special situations, as discussed below. In general they have no accessible noninteracting counterpart. There is then no natural set of single-particle modes and $\mathcal{E}_1$ lacks a comparable intuitive interpretation. It is a generalization of occupation number entanglement to such cases.

We now evaluate $\mathcal{E}_1$ for two typical electronic nanostructures. Since we are interested in their ground state entanglement we take the limit of zero temperature in all examples. First we consider an open quantum dot with charging interaction. We assume that the charging energy E_c of the dot is large and address the regime $\Gamma_j \ll \gamma \ll |\epsilon_j| \ll E_c$, $\tau^{-1} \approx \gamma$, and $v_j' \ll v_j, \tilde{v}_j$. We additionally require a small level spacing $\Delta \ll \Gamma_j$ and an in-plane magnetic field that renders the electrons effectively spinless. We probe the quantum dot at two points at a distance L from each other by two resonant levels 0 and 1 contacted with empty and occupied reservoirs respectively, as shown in Fig. 2. We choose $\epsilon_0 < 0$ and $\epsilon_1 > 0$ relative to the Fermi energy $\mu_{\mathrm{QD}} = 0$ of the dot. The quantum dot is opened by point contacts with a large total number of channels $N \gg 1$. The effects of the charging interaction can then be treated perturbatively within the incoherent model of quantum dots [21]. We find at $L = 0$ to leading order in our limits

$$\mathcal{E}_1^{\mathrm{QD}} = \frac{1}{N}\frac{\bar{\Gamma}}{\epsilon_1 - \epsilon_0}\left(\ln N\frac{\epsilon_1-\epsilon_0}{\bar{\Gamma}} + 1\right), \tag{8}$$

where $\bar{\Gamma}^{-1} = (\Gamma_0^{-1} + \Gamma_1^{-1})/2$. In this structure not only the leading order statistical correlations $\alpha^{\mathrm{QD}}_{\mathrm{nonint}} = -\Gamma_0\Gamma_1/(\epsilon_1-\epsilon_0)^2$, but also the interaction-induced correlations $\alpha^{\mathrm{QD}} = \bar{\Gamma}/N(\epsilon_1 - \epsilon_0)$ become small for $\bar{\Gamma} \ll \epsilon_1 - \epsilon_0$. The latter ones, however, dominate and thus unambiguously identify many-body interactions for $(\Gamma_0 + \Gamma_1)/2(\epsilon_1 - \epsilon_0) \ll 1/N$. This is the regime of validity of Eq. (8). It lies well within the range of typical experimental parameters. α^{QD} decays over distances $L \simeq \min\{v_F/\Gamma_j, l_{\mathrm{in}}\}$, where v_F is the Fermi velocity and l_{in} the inelastic length, and it acquires an oscillatory contribution at $L \simeq v_F/|\epsilon_1 - \epsilon_0|$ [18]. It persists up to temperatures of the order of $|\epsilon_j|$. Eq. (7) cannot be used to interpret $\mathcal{E}_1^{\mathrm{QD}}$ since the charging interaction cannot be

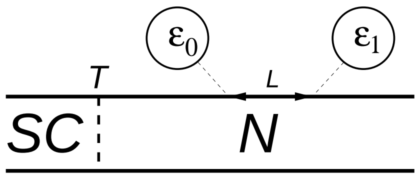

FIG. 3: Normal metal (N) tunnel-coupled (transmission $T \ll 1$) to a superconductor (SC).

switched off. Nevertheless, $\alpha^{\mathrm{QD}} \propto (\epsilon_1 - \epsilon_0)^{-1}$ reflects the typical energy-dependence of the probability of particle-hole excitations in effectively one-dimensional interacting electron systems. This illustrates that in particular the dependence of $\mathcal{E}_1$ on ϵ_0 and ϵ_1 contains valuable information about the ground state of an interacting fermion system. Ref. [22] reports an experiment on a setup very similar to the one shown in Fig. 2. The measurement of the proposed many-fermion entanglement in this structure should thus be experimentally feasible.

As a second example we consider a structure where interactions may effectively be switched off during the current measurement: a piece of normal metal with well-screened interactions weakly coupled to a BCS superconductor, as shown in Fig. 3. The superconductor induces pair-correlations in the normal metal through the so-called proximity effect that result in a nonzero many-fermion entanglement $\mathcal{E}_1$. If the measurement is performed in the noninteracting part of the structure at a sufficiently large distance from the superconductor, where the pairing interaction is present, one expects that $\mathcal{E}_1$ can be interpreted with the help of Eq. (7). We will confirm this intuition in Ref. [18]: for certain (not necessarily lowest energy) states one can show that the normalized correlator α^{SC} is indeed in precise correspondence with occupation number correlations in the spirit of Eq. (7). Also in this example much of the information content of α^{SC} is in its dependence on the level energies ϵ_j: α^{SC} is maximal for $\epsilon_0 = -\epsilon_1$ (the Fermi energy of the superconductor is chosen $\mu_{\mathrm{SC}} = 0$) and it decays for $|\epsilon_0 + \epsilon_1| > \Gamma_j$ [18].

Other entanglement measures based on current correlations in nanostructures have been put forward in Refs. [23]. These entanglement measures are designed to quantify two-particle correlations. They do not distinguish interaction-induced correlations from statistical ones. The entanglement captured by these measures is present also in certain noninteracting fermion systems [24]. This contrasts with the many-fermion entanglement proposed here which vanishes in noninteracting ground states. It singles out correlations due to many-body interactions.

In conclusion, we have proposed a measure of many-fermion entanglement based on a generalized notion of entanglement developed in Refs. [3, 4]. It quantifies ground state correlations that are induced by many-body interactions and it is experimentally accessible through cross-correlation measurements of currents through resonant fermion levels. It measures a generalization of occupation number entanglement [6, 7]. With two examples we have illustrated the introduced entanglement measure and highlighted aspects of interaction-induced ground state correlations that it serves to characterize. The experimental techniques that its measurement requires are all existing in electronic nanostructures. It thus promises to be a systematic way of studying interacting fermion systems, both, theoretically as well as experimentally.

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